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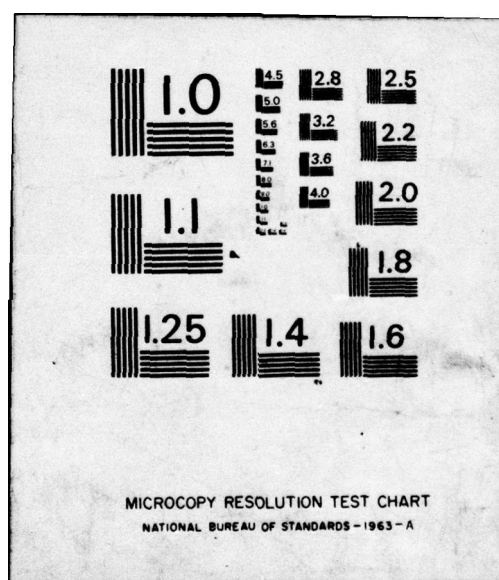
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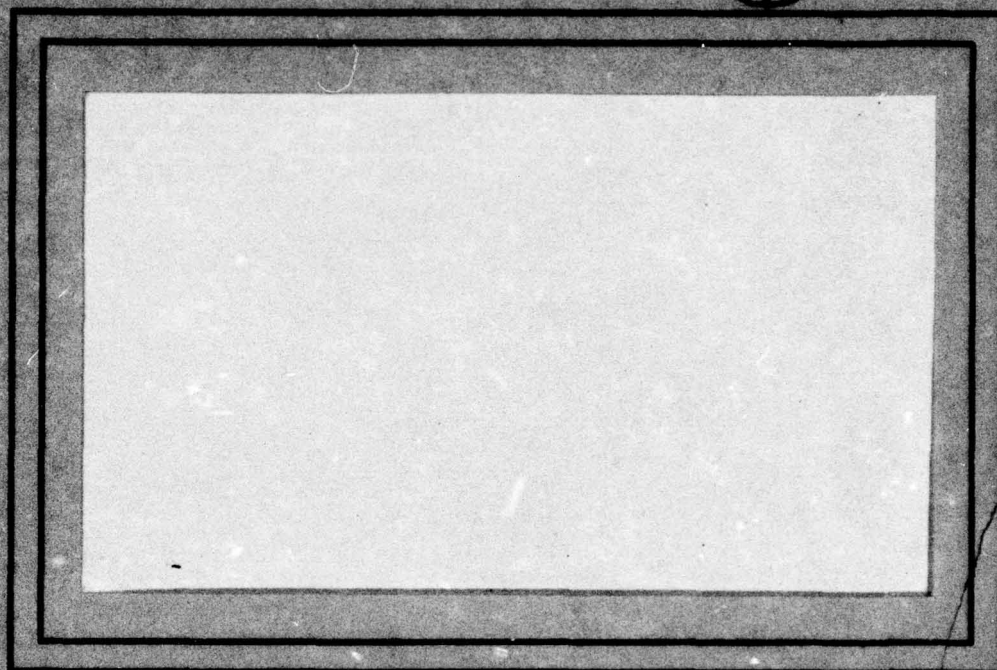
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SOME NEW METHODS OF

DETECTING STEP EDGES

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ABSTRACT

This note describes three operators that respond to step
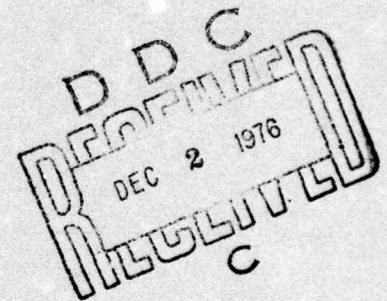edges, but not to ramps. The first is similar to the digital
Laplacian, but uses the max, rather than the sum, of the x and
y second differences. The second uses the difference between
the mean and median gray levels in a neighborhood as an edge
value at the center of the neighborhood. The third method com-
putes moments of inertia of the neighborhood about a set of
lines through its center, and takes the difference between the
greatest and smallest of these as the edge value at the center.
The outputs obtained from these operators applied to a set of
test pictures are compared with each other and with the standard
digital Laplacian and gradient. A fourth operator, which uses
the distance between the center and centroid of a neighborhood
as an edge value, is also briefly considered; it turns out to be
equivalent to one of the standard digital approximations to the
gradient.

1.   Introduction

Many edge detection techniques have been described in the literature on image processing and pattern recognition; see [1-2] for reviews. This note describes several new edge detection operators. Like the digital Laplacian (see Section 2), these operators respond to steps, but not to ramps; and two of them seem to be less sensitive to noise than the Laplacian. They are all simple to compute, and under some circumstances might even be computationally advantageous -- e.g., if one were doing median filtering, the mean-median difference operator (Section 3) might be a preferred edge detector.

The first operator, which we call the pseudoLaplacian, is a minor variation on the standard digital Laplacian, but appears to be less noise-sensitive; it is described in Section 2. The second operator uses the difference between the mean and median gray levels in a neighborhood as an edge value; it is quite insensitive to noise (Section 3). The third operator computes the moments of inertia of a neighborhood about lines through its center, and takes the greatest minus the smallest of these moments as an edge value (Section 4). A fourth operator, which uses the distance between the center and centroid of a neighborhood as an edge value, is discussed in Section 5; it turns out to be mathematically equivalent to one of the standard digital approximations to the gradient.

2. The pseudoLaplacian

Two derivative operators that have commonly been used
as edge detectors are the magnitude of the gradient

$$\sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}$$

and the Laplacian

$$\frac{\partial^2 f}{\partial x^2} \qquad \frac{\partial^2 f}{\partial y^2}$$

For digital pictures, finite differences are used instead
of derivatives, i.e.,

$$\Delta_x f = f(x+1,y) - f(x,y) \quad \text{for} \quad \frac{\partial f}{\partial x}$$

and

$$\Delta^2_x f = f(x+1,y) + f(x-1,y) - 2f(x,y) \quad \text{for} \quad \frac{\partial^2 f}{\partial x^2}$$

and similarly for the y-derivatives. For computational
simplicity, in the digital "gradient", the sum or max of
$|\Delta_x f|$ and $|\Delta_y f|$ are often used instead of the square root
of the sum of their squares.

In general, the gradient is a better edge detector
than the Laplacian, because the latter responds more
strongly to isolated points than it does to edges. Indeed,
it is easily verified that the response of both operators
to a step edge of unit height is 1; but for an isolated
spike of unit height, the gradient response is 1 while the
Laplacian response is 4. The gradient also responds to
ramps, while the Laplacian response to a ramp is zero.

We define the pseudoLaplacian as the max, rather than the sum, of the second differences, i.e.,

$$\max(\Delta^2_x f, \Delta^2_y f)$$

Readily, this has response 2, rather than 4, to a spike of unit height, and still has zero response to a ramp.

Figures 1 and 2a-b compare the responses of several of these operators applied to a set of test pictures. (Some of these pictures were also used in [2] to illustrate the performance of other edge detector techniques.) The operators tested are:

1) The digital "gradient" $\max(|\Delta_x f|, |\Delta_y f|)$

2) The digital "gradient" $|\Delta_x f| + |\Delta_y f|$

3) The positive digitial Laplacian $\max[0, \Delta^2_x f + \Delta^2_y f]$

4) The absolute digital Laplacian $|\Delta^2_x f + \Delta^2_y f|$

5) The positive pseudoLaplacian $\max[0, \Delta^2_x, \Delta^2_y f]$

6) The absolute pseudoLaplacian $\max[|\Delta^2_x f|, |\Delta^2_y f|]$

Note that for a horizontal or vertical step edge of unit height, these operators all have value $+1$; their outputs have therefore all been scaled alike for display. (Specifically, the values have all been multiplied by 2, with values beyond the top of the grayscale truncated to the maximum gray level.)

For diagonal edges, the operators based on sums have twice the response of those based on maxima; this can be seen by comparing Figs. 1b, c, d with Figs. 1a, e, f. The "absolute" operators (Figs. 1c, e) yield thicker edges than

the "positive" operators (Figs. 1d, f), since they detect edges in two positions. For example, the absolute Laplacian has value 1 at both of the underlined points adjacent to the edge ..00̲1̲1..., whereas the positive Laplacian has value 1 only at the underlined 0̲, since the Laplacian is negative at the underlined 1̲.

Comparison of Figs. 1d and 2b indicates that the absolute pseudoLaplacian outputs are at least as noisy as the absolute Laplacian outputs, but they respond somewhat better to blurred edges (e.g., the large chromosomes in the third column), though not as strongly as the gradient operators (Fig. 1a-b). On the other hand, comparison of Figs. 1c and 2a shows that the outputs of the positive pseudoLaplacian appear to be appreciably less noisy than those of the positive Laplacian.

3. The mean-median difference

Suppose that a point is just adjacent to a step edge, so that its 3-by-3 neighborhood looks like (e.g.)

```
zzw            zww
zzw   or       zzw        (or rotations of these).
zzw            zzz
```

Then the mean μ of the gray levels in such a neighborhood is $(6z+3w)/9 = \frac{2}{3}z + \frac{1}{3}w$, whereas the median m of these gray levels is z. The mean and median thus differ by $z - (\frac{2}{3}z + \frac{1}{3}w) = \frac{1}{3}(z-w)$, which is proportional to the contrast of the edge. Note that for a linear ramp, e.g., $\begin{matrix}123\\123\\123\end{matrix}$, the mean and median are the same (=2); thus this edge detection operator, like the Laplacian, responds to steps but not to ramps. For an isolated noise point $\begin{matrix}zzz\\zwz\\zzz\end{matrix}$ the median is z, while the mean is close to z (namely, $\frac{8z+w}{9} = z + (w-z)/9$), so that the mean-median difference is only a third as great as it is for a step edge. Thus this operator should be quite insensitive to noise.

Figure 2c-d show positive and absolute mean-median differences, i.e., $\max[0,m-\mu]$ and $|m-\mu|$, for the same pictures as in Figure 1. As before, the absolute differences yield thicker edges than the positive difference. Since the response of these operators to a step edge of unit height is only $\frac{1}{3}$, the output values have been scaled by a factor of 3 relative to the values shown in Figure 1. The responses are markedly less noisier than those of the Laplacians.

4.   The moment range

Suppose once again that a point is just adjacent to a
step edge, and has a 3x3 neighborhood which is a rotation
of

```
zzw              zww
zzw     or       zzw
zzw              zzz
```

The moments of inertia of these neighborhoods about various
lines through their centers are as follows:

	Moment of inertia	
	zzw	zww
Line slope	zzw	zzw
	zzw	zzz
0	4z+2w	4z+2w
45	4z+2w	4z+2w
90	3z+3w	4z+2w
135	4z+2w	3z+3w

(Note that we have treated all points of the neighborhood
not lying on the line as being at distance 1 from the
line; this amounts to taking moments with respect to chess-
board distance, in which all eight neighbors of a point are
at distance 1 from the point.) Thus in each case, the
greatest and least moments have values 4z+2w and 3z+3w;
which of these is greater than the other depends on whether
z > w or z < w. In either event, the absolute difference
between the greatest and least moments -- i.e., the range
of the moment values -- is just |z-w|, which is the con-
trast of the edge.

Here again, for a linear ramp, e.g., $\begin{smallmatrix}123\\123\\123\end{smallmatrix}$, the moments
are all equal; thus we have once again defined an operator
that responds to steps but not to ramps. (For an isolated
noise point $\begin{smallmatrix}zzz\\zwz\\zzz\end{smallmatrix}$ the moments are all equal (to 6z), so that
their range is zero, implying that this operator should be
insensitive to noise; but this is not borne out by our examples.)

Figure 2f shows absolute moment range values for the
same pictures as in Figure 1. There is no obvious analog
of a "positive" moment range operator, since the range is
always nonnegative; but we can force the moment range re-
sponses to be thinner by making the operator amisotropic.
Specifically, in the neighborhood

$$abc$$
$$def$$
$$ghi$$

let $m_v = a+b+c+g+h+i$ $\qquad$ $m_d = a+b+d+f+h+i$

$\qquad$ $m_h = a+d+g+c+f+i$ $\qquad$ $m_d' = b+c+f+d+h+g$

Then we can define a "positive" moment range as, e.g.,

$$\max[0, \max(m_v, m_d) - \min(m_h, m_d')]$$

The outputs of this operator for the test pictures are
shown in Figure 2e. The results are indeed thinner than
those in Figure 2f, but the shapes of the objects in the
pictures are somewhat distorted (e.g., see the first picture
of the set). These outputs are scaled the same as those of
Figure 1, since the response to a step edge of unit height
is unity here also.

5. The center-centroid distance

We conclude by describing another edge detection opera-
tor whose definition also involves moments, but which turns
out to be equivalent to one of the standard digital gradient
operators. The idea for this operator was suggested by a
method used by Zucker [in preparation] to detect the edges
of dot clusters.

In the neighborhood

$$abc$$
$$def$$
$$ghi$$

if we take the center e of the neighborhood as origin, then
the coordinates of the neighborhood's centroid (ignoring
the scale factor a+b+c+d+e+f+g+h+i) are

$$m_x = (c+f+i)-(a+d+g)$$
$$m_y = (a+b+c)-(g+h+i)$$

These are just the x- and y-components of a commonly used
digital gradient operator [2] which combines smoothing along
the edge with differencing across it. Thus the distance
between the center of the neighborhood and its centroid,
i.e.,

$$\sqrt{m^2_x+m^2_y} \qquad \text{(Euclidean distance)}$$

$$\text{or} \qquad |m_x|+|m_y| \qquad \text{(City block distance)}$$

$$\text{or} \qquad \max(|m_x|,|m_y|) \qquad \text{(Chessboard distance)}$$

can be used as an edge measure, since it is an approximation

to the gradient magnitude. Note that this measure does re-
spond to ramps, since it is based on first rather than second
differences. A simpler operator can be defined using 2-by-2
neighborhoods; in fact, for $\begin{smallmatrix} ab \\ cd \end{smallmatrix}$ the coordinates of the
centroid relative to the center are proportional to
(b+d)-(a+c) and (a+b)-(c-d), respectively, which are the
components of another standard gradient approximation (see
[2], p. 285, Fig. 13).

6. Concluding remarks

This note suggests that many simple variations on the standard edge detection operators are possible, and in particular, that one can design Laplacian-like operators (e.g., the positive mean-median difference)   much less sensitive to noise than the standard Laplacian. It is hoped that these operators will find their place among the growing array of tools that are becoming available for image processing and analysis.

References

1. L. S. Davis, A survey of edge detection techniques, Computer Graphics Image Processing 4, 1975, 248-270.

2. A. Rosenfeld and A. C. Kak, Digital Picture Processing, Academic Press, New York, 1976, Section 8.2.
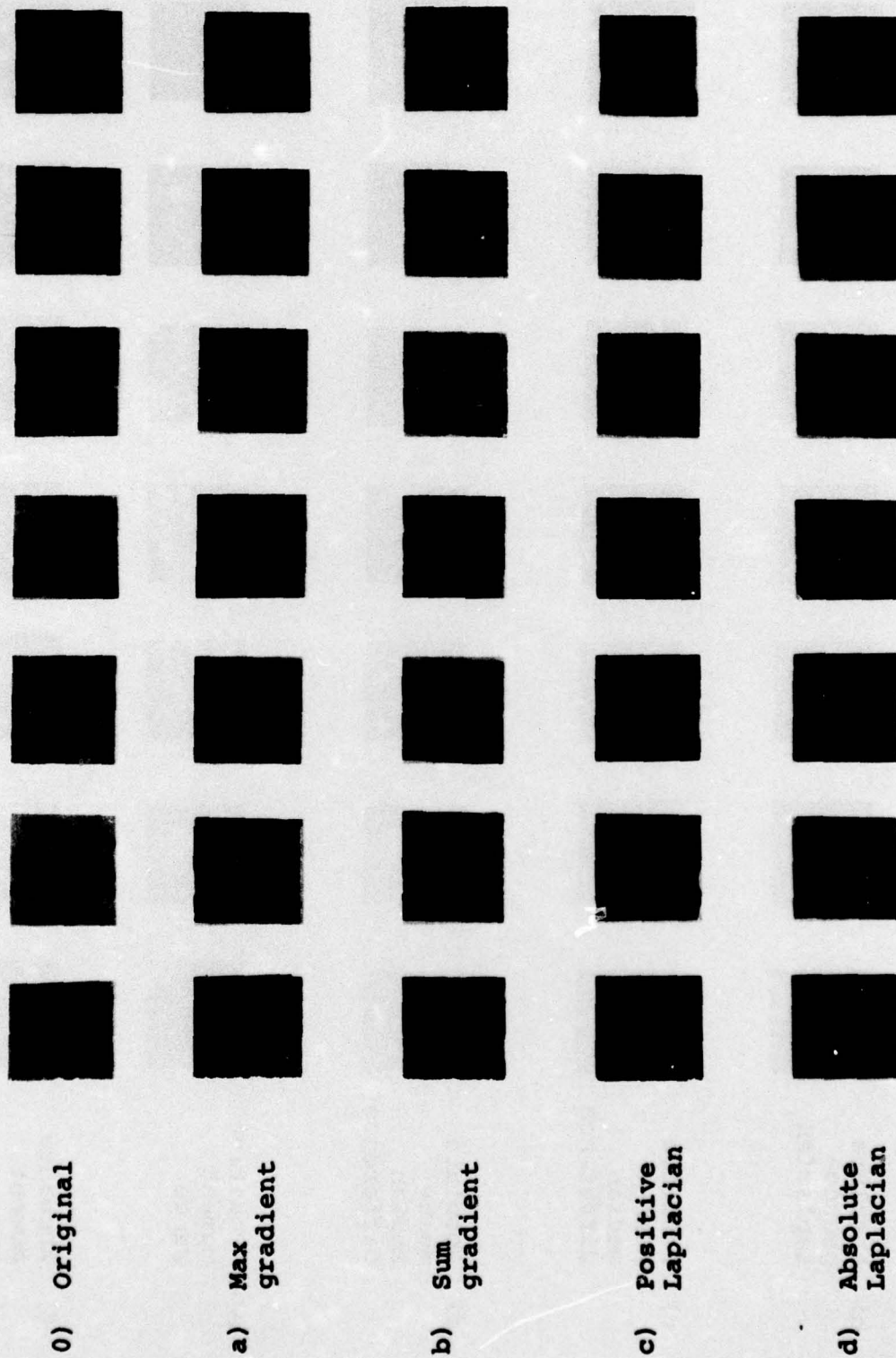
0) Original

a) Max gradient

b) Sum gradient

c) Positive Laplacian

d) Absolute Laplacian

Figure 1. Digital gradients and Laplacians.

a) Positive
    pseudo-
    Laplacian

b) Absolute
    pseudo-
    Laplacian

c) Positive
    mean-
    median
    difference

d) Absolute
    mean-
    median
    difference

e) "Positive"
    moment
    range

f) Absolute
    moment
    range
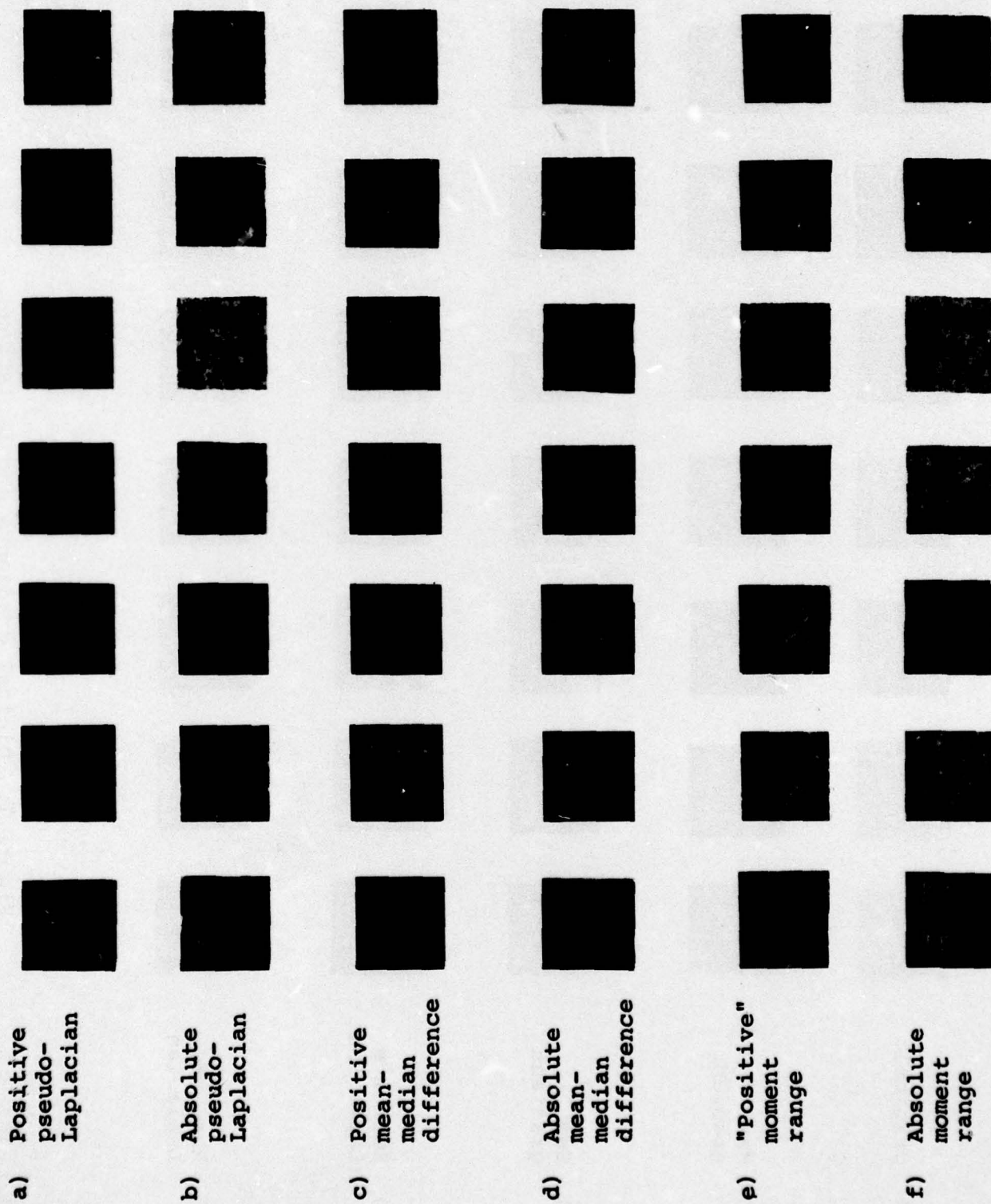
Figure 2. PseudoLaplacians, mean-median differences, and moment ranges.

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Edge detection   Moments
Laplacian   Centroid
Gradient

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Abstract (continued)

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